WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 5th Semester Examination, 2022-23

MTMGDSE01T-MATHEMATICS (DSE1)

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any *five* questions from the following: $2\times 5 = 10$

(a) Is the set of vectors (1, 0, 0), (0, 1, 0) and (0, 0, 1) are linearly dependent? Justify your answer. 1+1

(b) What is the geometric meaning of the given transformation

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

(c) Is any straight line passing through (0, 0, 0) in $\mathbb{R}^3$ a sub space of $\mathbb{R}^3$? Give reason.

(d) Write down the matrix form of the system of equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

(e) Is the vectors (1, 2) and (–1, 2) and linearly independent in $\mathbb{R}^3$? Justify.

(f) Find the inverse of the matrix $\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$.

(g) For what values of k the three vectors (1, 2, 2), (k, 1, 2) and (2, 2, 1) are linearly independent?

(h) Write the standard basis of $\mathbb{R}^2$ and $\mathbb{R}^3$.

(i) Prove that $S = \{(x, y, z) \in \mathbb{R}^3 / x + y + z = 0\}$ is a subspace of $\mathbb{R}^3$.

2. (a) If u and v are linearly independent vectors in a vector space V then show that so are $u+v$ and $u-v$. 2

(b) Examine whether the set of vectors are linearly independent in $\mathbb{R}^3$. 4

$$\{(1, 2, 3), (2, 3, 1), (3, 1, 2)\}.$$

(c) Define Dilation and Rotation. 2

3. (a) Let A be a singular matrix. Is 0 is an eigen value of A? Justify your answer. 4

(b) If λ be an eigen value of a non-singular matrix A, then λ^{-1} is an eigen value of A^{-1}. 4

4. (a) Define Eigen space and invariant space with examples. 2+2

(b) Show that the matrix $A=\begin{pmatrix}1 & 0\\ 3 & 1\end{pmatrix}$ is not diagonalisable. 4

5. (a) Define a basis of a vector space. Do the vectors (1, 0, 0), (0, 1, 0) and (1, 2, 1) are form a basis of $\mathbb{R}^3$? Justify. 4

(b) Find the eigen values and corresponding eigen vectors of the following real matrix. 4

$$\begin{pmatrix}2 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 5\end{pmatrix}$$

6. (a) Prove or disprove: The set $\{(x, y, z)\in \mathbb{R}^3 \mid ax+by+cz=0 \text{ and } a^2+b^2+c^2\neq 0\}$ is a subspace of $\mathbb{R}^3$. 3

(b) Use elementary row operations on A to obtain A^{-1} where 5

$$A=\begin{bmatrix}2 & 0 & 0\\ 4 & 3 & 0\\ 6 & 4 & 1\end{bmatrix}$$

7. (a) Let $A=\begin{bmatrix}3 & 2 & -6\\ 0 & -1 & 4\\ 5 & -2 & 0\end{bmatrix}$. Verify that $A+A'$ is symmetric and $A-A'$ is skew-symmetric and hence express A as the sum of a symmetric and skew-symmetric matrix. 4

(b) If $A=\begin{bmatrix}1 & 0 & 2\\ 0 & -1 & 1\\ 0 & 1 & 0\end{bmatrix}$, then verify that A satisfies its own characteristic equation. Hence find A^{-1} and A^9. 4

8. (a) Find a basis of $\mathbb{R}^3$ containing the vectors (1, 1, 0) and (1, 1, 1). 3

(b) Let A, B, C be three square matrices such that $A\neq O$ and $AB=AC$, where O is the null matrix. Does it imply $B=C$? Justify your answer. 3

(c) Define dimension of a finite dimensional vector space V over the field F. Give example. 2

9. (a) Find the 3 by 3 matrix representations of the following transformations. 2+2
   (i) projection of any point on the x-y plane.
   (ii) reflection of any point through the x-y plane.

   (b) Determine the rank of $A=\begin{pmatrix} x & 1 & 0 \\ 3 & x-2 & 1 \\ 3(x+1) & 0 & x+1 \end{pmatrix}$, for different values of x. 4

10. (a) Solve by matrix method: 4

$$x+y+z=4,$$
$$2x-y+3z=1,$$
$$3x+2y-z=1.$$

   (b) Reduce the matrix to fully reduced normal form 4

$$\begin{pmatrix} 1 & 0 & 2 & 3 \\ 2 & 0 & 4 & 6 \\ 3 & 0 & 7 & 2 \end{pmatrix}.$$

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WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 5th Semester Examination, 2021-22

MTMGDSE01T-MATHEMATICS (DSE1)

Time Allotted: 2 Hours | Full Marks: 50

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Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Express $v=(x, y)$ as a linear combination of $v_1=(1,1)$ and $v_2=(1,-1)$ in $\mathbb{R}^2$.

(b) What 2 by 2 matrices represent the transformations that

(i) rotate every point by an angle θ about the origin.

(ii) reflect every point about the x-axis.

(c) What is the geometric object corresponding to the smallest subspace V_0 containing a nonzero vector $v=(r, s, t)\in\mathbb{R}^3$? Answer with reason.

(d) Write the matrix equation for the system of equations:

$$x+y=3,\ -3y+4z=17,\ x-z=-8.$$

(e) Is there any straight line in the vector space R_2 which is a subspace of R_2?

(f) Find the inverse of the matrix $A=\begin{bmatrix} 5 & 3 \\ -2 & 2 \end{bmatrix}$.

(g) For what values of z the three vectors (1, 1, 2), (z, 1, 1) and (1, 2, 1) are linearly independent?

(h) It is impossible for a system of linear equations to have exactly two solutions. Explain why.

(i) Prove that $\{(x, y, z)\in\mathbb{R}^3 : x^2+y^2=z^2\}$ is not a subspace of $\mathbb{R}^3$.

2. (a) Examine if the set S is a subspace of $\mathbf{R}_3$, $S=\{(x, y, z)\in \mathbf{R}_3 \mid x=0\ z=0\}$. 4

(b) If $\alpha=(1, 2, 0)$, $\beta=(3, -1, 1)$, and $\gamma=(4, 1, 1)$, determine whether they are linearly dependent or not. 4

3. (a) If $A=\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ $B=\begin{bmatrix} p & q & r \\ s & t & u \end{bmatrix}$, $C=\begin{bmatrix} l & m \\ n & k \\ h & g \end{bmatrix}$, then establish that $(A+B)C = AC+BC$. 4

(b) If $P=\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, and $Q=\begin{bmatrix} p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \\ r_1 & r_2 & r_3 \end{bmatrix}$ then establish 4

(i) $(P+Q)^T = P^T + Q^T$ and (ii) $(P.Q)^T = Q^T.P^T$.

4. (a) Prove that two eigen vectors of a square matrix A over a field F corresponding to two distinct eigen values of A are linearly independent. 4

(b) Prove that the eigen values of a real symmetric matrix are all real. 4

5. (a) Prove that a matrix is non-singular if and only if it can be expressed as the product of a finite number of elementary matrices. 4

(b) Prove that if the rank of a real symmetric matrix be 1 then the diagonal elements of the matrix cannot be all zero. 4

6. (a) Diagonaliza the matrix $A=\begin{bmatrix} 6 & 4 & -2 \\ 4 & 12 & -4 \\ -2 & -4 & 13 \end{bmatrix}$. 5

(b) Define a basis of a vector space. Do the vectors (1, 1, 2), (3, 5, 2) and (1, 0, 0) form a basis of $\mathbb{R}^3$? Justify. 3

7. (a) Find the eigen vectors and eigenvalues of $\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$ 4

(b) If Q_θ represents the matrix for rotation (in x-y plane) through an angle θ about the origin, prove that $Q_\theta^2 = Q_{2\theta}$ and $Q_\theta Q_{-\theta} = I_2$ 4

8. (a) State Cayley-Hamilton's Theorem and verify it for the matrix $A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. 4

Hence find A^{-1}.

(b) What matrix has the effect of rotating every point through 90º and then projecting the result onto the x-axis? What matrix represents projection onto the x-axis followed by projection onto y-axis? 2+2

9. (a) Determine the rank of the matrix $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 8 & 6 \\ 0 & 0 & 5 & 8 \\ 3 & 6 & 6 & 3 \end{bmatrix}$. 4

(b) If $A=\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$, $C=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Correct or justify: 4

(i) $(A-B)(A+B)=A^2-B^2$

(ii) $(A-C)(A+C)=A^2-C^2$

10.(a) Express $A=\begin{bmatrix} 2 & 5 & -3 \\ 7 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ as a sum of a symmetric and skew symmetric matrix. 3

(b) If $A=\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $C=\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ then verify that $AC=CA=6I_3$ and use this result to solve the system of equations 5

$x-y=3,\ 2x+3y+4z=17,\ y+2z=7$.

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WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 5th Semester Examination, 2021-22

MTMGDSE02T-MATHEMATICS (DSE1)

MECHANICS

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates should answer in their own words and adhere to the word limit as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: 2×5 = 10

(a) Write down the conditions of equilibrium of a system of coplanar forces acting on a rigid body.

(b) Three forces *P*, *Q*, *R* act in the same sense along the sides $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ of a triangle *ABC*. If their resultant passes through the in-centre then show that $P+Q+R=0$.

(c) Find the centre of gravity of the area bounded by the parabola $y^2=4ax$ and its latus rectum.

(d) A heavy body is in limiting equilibrium on a rough inclined plane under the action of gravity only, then what is the inclination of the plane with the horizontal?

(e) A particle moves along a straight line according to the law $s^2=at^2+bt+c$. Prove that its acceleration varies as $\frac{1}{s^3}$.

(f) At what height would the kinetic energy of a falling particle be equal to half of its potential energy?

(g) If a particle moves in a circle of radius *r* with uniform speed *v*, then find its angular velocity about the centre.

(h) A particle is projected under gravity at an angle α with the horizontal. Find the velocity of the particle at time *t*.

(i) A particle describes the curve $r=ae^{\theta}$ with constant angular velocity. Show that its radial acceleration is zero and the transverse acceleration varies as the distance from the pole.

2. Three forces *P*, *Q*, *R* act along the sides of the triangle formed by the lines $x+y=1$, $y-x=1$ and $y=2$. Find the equation of the line of action of their resultant. 8

3. Show that the least force which will move a weight W along a rough horizontal plane is $W\sin\phi$, where ϕ is the angle of friction. 8

4. A frustum of a cone is formed by cutting off the upper portion of a solid right circular cone by a plane parallel to the base. The radii of the parallel circular sections being R and r, and h the height of the frustum, show that the height of the centre of gravity of the frustum from the base is $\frac{h}{4}\cdot\frac{R^2+2Rr+3r^2}{R^2+Rr+r^2}$. 8

5. (a) A cycloid is placed with its axis vertical and vertex downwards. Show that a particle cannot rest at any point of the curve which is higher than $2a\sin^2\lambda$ above the lowest point, where λ is the angle of friction and a is the radius of the generating circle of the cycloid. 3

(b) Two equal uniform rods AB and AC, each of length $2b$ are freely jointed at A and rest on a smooth vertical circle of radius a. Show that if 2θ be the angle between them, then $b\sin^3\theta = a\cos\theta$. 5

6. (a) Deduce the expressions for tangential and normal components of the acceleration of a particle describing a plane curve. 5

(b) A particle describes a circle of radius a in such a way that its tangential acceleration is K times the normal acceleration, where K is a constant. If the speed of particle at any point be u, prove that it will return to the same point after a time 3

$$\frac{a}{Ku}(1-e^{-2\pi K})$$

7. Two particles are projected simultaneously from O in different directions with same speed u so as to pass through another point P. If α and β are the angles of projection prove that they pass through P at times separated by 8

$$\frac{2u}{g}\sin\frac{1}{2}(\alpha-\beta)\cdot\sec\frac{1}{2}(\alpha+\beta)$$

8. (a) A particle of mass m falls from rest at a distance a from the centre of force varying inversely as the square of the distance from the centre. Find the time it descends to the centre of force. 4

(b) A particle moving in a straight line starts from rest and the acceleration at any time t is $a-Kt^2$, where a and K are positive constant. Show that the maximum velocity attained by the particle is $\frac{2}{3}\sqrt{\frac{a^3}{K}}$. 4

9. (a) A particle rests in equilibrium under the attraction of two centre of force which attract directly as the distance, their attractions at unit distance being μ_1 and μ_2 respectively. The particle is slightly displaced towards one of the centres, show that the time of small oscillation is $\frac{2\pi}{\sqrt{\mu_1+\mu_2}}$. 4

(b) In a simple harmonic motion, if f be the acceleration and v be the velocity at any instant and T is periodic time, then show that $f^2T^4 + 4\pi^2v^2T^2 = 16\pi^4a^2$. 4

10.(a) A particle is projected vertically upwards with a velocity u in a medium whose resistance varies as the square of the velocity. Investigate the motion. 4

(b) If the radial and transverse velocities of a particle are $\mu\theta$ and λr respectively, show that the path of the particle can be represented by an equation of the form $r = A\theta^2 + B$. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 5th Semester Examination, 2020, held in 2021

MTMGDSE01T-MATHEMATICS (DSE1)

MATRICES

Time Allotted: 2 Hours | Full Marks: 50

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All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times5 = 10$

(a) Write short note on Linear independence of vectors.

(b) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 5 & 2 \\ 4 & 8 & 0 \end{bmatrix}$.

(c) Show that for two non-singular matrices A and B of same order $(AB)^{-1} = B^{-1}.A^{-1}$.

(d) State Cayley-Hamilton's theorem.

(e) Show that the matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ is an orthogonal matrix.

(f) Show that $S = \{(x, y, z) \in \mathbb{R}^3 : 3x - 4y + z = 0\}$ is a sub-space in $\mathbb{R}^3$.

(g) Show that the vectors $\alpha_1 = (0, 2, -4)$, $\alpha_2 = (1, -2, -1)$, $\alpha_3 = (1, -4, 3)$ are linearly dependent.

(h) Write a simple 3×3 matrix whose all eigen values are 1, 2, 3 respectively.

(i) When a matrix is not invertible?

(j) Write the equations in matrix from $x_1 = x\cos\alpha + y\sin\alpha$ and $y_1 = -x\sin\alpha + y\cos\alpha$.

2. (a) Examine whether the set S is a subspace of $\mathbf{R}_3$ or not, where 4

$$S = \{(x,\ y,\ z) \in \mathbf{R}_3 \mid x = 0\}$$

(b) If $\alpha = (1,\ 1,\ 2)$, $\beta = (0,\ 2,\ 1)$, and $\gamma = (2,\ 2,\ 4)$, determine whether they are linearly independent or not. 4

3. (a) If $A=\begin{bmatrix}1 & 3 & 0\\ 3 & 7 & 2\end{bmatrix}$, $B=\begin{bmatrix}2 & 5 & 8\\ 1 & 3 & 2\end{bmatrix}$, $C=\begin{bmatrix}1 & 3\\ 2 & 4\\ 1 & 5\end{bmatrix}$, then establish that $(A+B)C=AC+BC$. 4

(b) If $P=\begin{bmatrix}6 & 12 & 13\\ 14 & 24 & 25\\ 10 & 16 & 18\end{bmatrix}$, and $Q=\begin{bmatrix}11 & 8 & 3\\ 13 & 9 & 15\\ 14 & 21 & 18\end{bmatrix}$ then establish 4

(i) $(P+Q)^T=P^T+Q^T$ and (ii) $(P.Q)^T=Q^T.P^T$

4. (a) Find a basis and the dimension of the subspace W of $\mathbb{R}^3$, where 3+1

$$W=\{(x,\ y,\ z)\in R^3\ :\ x+y+z=0\}$$

(b) If $A+I=\begin{bmatrix}1 & 3 & 4\\ -1 & 1 & 3\\ -2 & -3 & 1\end{bmatrix}$, evaluate $(A+I)(A-I)$, where I represents the 3×3 identity matrix. 4

5. (a) Find the inverse of the matrix $\begin{bmatrix}2 & 1 & 1\\ 1 & -1 & 0\\ 2 & 1 & -1\end{bmatrix}$ and use it to solve the following system of equations: 2+2

$$2x+y+z=5$$
$$2x+y-z=1$$
$$x-y=0$$

(b) Solve by matrix method: 4

$$2x-y=1$$
$$x+y=2$$

6. (a) Find the eigen values of the matrix $\begin{bmatrix}1 & -1 & 2\\ 1 & 1 & 1\\ 1 & 3 & 1\end{bmatrix}$. 4

(b) Prove that if λ be an eigen value of a non-singular matrix A, then λ^{-1} is an eigen value of A^{-1}. 4

7. (a) Prove that two eigen vectors of a square matrix A over a field F corresponding to two distinct eigen values of A are linearly independent. 4

(b) Prove that the eigen values of a real symmetric matrix are all real. 4

8. (a) Find the rank of the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 3 & 4 & 5\\ 4 & 6 & 8\end{bmatrix}$. 4

(b) Define elementary matrix. Also show that elementary matrices are non singular. 4

9. (a) Prove that a matrix is non-singular if and only if it can be expressed as the product of a finite number of elementary matrices. 4

(b) Prove that if the rank of a real symmetric matrix be 1 then the diagonal elements of the matrix cannot be all zero. 4

10.(a) Use Cayley-Hamilton theorem to find A^{100}, where $A=\begin{bmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$. 4

(b) Show that $B=\{(1,\ 2,\ 1),\ (0,\ 1,\ 0),\ (0,\ 0,\ 1)\}$ is a basis of $\mathbb{R}^3$. Express the vector $(1,\ 2,\ 3)\in\mathbb{R}^3$ as a linear combination of the basis B. 1+3

11.(a) Reduce the matrix to the fully reduced normal form 4

$$\begin{bmatrix}1 & 0 & 2 & 3\\ 2 & 0 & 4 & 6\\ 3 & 0 & 7 & 2\end{bmatrix}$$

(b) Find all real matrices $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, such that $A^2=I_2$. 4

12.(a) If $A=\begin{bmatrix}a & b & c & d\\ -b & a & d & -c\\ -c & -d & a & b\\ -d & c & -b & a\end{bmatrix}$, compute AA^t. 4

(b) Find matrix A, if $\operatorname{adj} A=\begin{bmatrix}1 & 3 & -4\\ -2 & 2 & -2\\ 1 & -3 & 4\end{bmatrix}$. 4

13.(a) Find the equation of the line through the following pair of points in (3, 7, 2) and (3, 7, –8). 2

(b) Find the equation of the plane containing the following point in space: 2

$$(1,\ 1,\ 1),\ (5,\ 5,\ 5) \quad \text{and} \quad (-6,\ 4,\ 2)$$

(c) Prove that the set $S=\{(1,\ 0,\ 1),\ (0,\ 1,\ 1),\ (1,\ 1,\ 0)\}$ is a basis of $\mathbb{R}^3$. 4

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WEST BENGAL STATE UNIVERSITY

B.Sc. Programme 5th Semester Examination, 2020, held in 2021

MTMGDSE02T-MATHEMATICS (DSE1)

MECHANICS

Time Allotted: 2 Hours | Full Marks: 50

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Answer Question No. 1 and any *five* from the rest

1. Answer any ***five*** questions from the following: $2\times 5 = 10$

(a) State Parallelogram Law of forces.

(b) If the resultant of two forces acting on a particle be at right angles to one of them, and its magnitude be one third of the magnitude of the other, then the ratio of the larger force to the smaller is $3:2\sqrt{2}$.

(c) Define limiting friction.

(d) If the masses of 3, 4, 5, 6 and 7 are placed at the four angular points A, B, C, D and the centre O respectively of a square of sides $2a$, then find the centre of gravity of the system.

(e) State Lami's theorem.

(f) What is astatic centre?

(g) A particle describes a s.h.m. in a straight line with amplitude 2 cms. Its velocity while passing the centre of oscillation is 12 cm/sec. Find its time-period.

(h) For a particle moving in a plane curve, if the tangential and normal accelerations are equal in magnitude then show that the velocity varies as e^{ψ}.

(i) A particle moves in a straight line with an acceleration always directed towards a fixed point on it and proportional to its distance from it in a medium which offers a small resistance proportional to its velocity. Write down the equation of motion of the particle.

(j) If the redial velocity of a particle is proportional to its cross-radial velocity, find the path in polar coordinates.

2. (a) Two forces of magnitudes $3P$, $2P$ respectively have resultant R. If the first force is doubled the magnitude of the resultant is doubled. Find the angle between the forces. 3

(b) Three forces P, Q, R acting along $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ are in equilibrium. If O be the circumcentre of the triangle ABC, then prove that 5

$$\frac{P}{\frac{1}{b^2}+\frac{1}{c^2}-\frac{a^2}{b^2c^2}}=\frac{Q}{\frac{1}{c^2}+\frac{1}{a^2}-\frac{b^2}{c^2a^2}}=\frac{P}{\frac{1}{a^2}+\frac{1}{b^2}-\frac{c^2}{a^2b^2}},$$

where a, b, c are the length of the sides BC, CA, AB respectively.

3. (a) A sphere of weight W is in equilibrium on a smooth plane of inclination α to the horizontal, being supported by a string, which is of length equal to the radius and is fastened to two points, one on the sphere and one on the plane. Prove that the tension of the string is $\frac{2}{3}\sqrt{3}W\sin\alpha$. 4

(b) A heavy uniform beam rests with its extremities on two smooth inclined planes. The inclination of the inclined planes to the horizontal are α and β. Find the inclination of the beam with the horizontal in the position of equilibrium and the reaction of the inclined planes. 4

4. (a) Forces P, Q, R act respectively along the lines $x=0$, $y=0$ and $x\cos\alpha+y\sin\alpha=p$. Find the magnitude of the resultant and the equation of its line of action. 4

(b) A solid hemisphere of weight W rests in limiting equilibrium with its curved surface on a rough inclined plane and the plane face is kept horizontal by a weight P attached to a point in the rim. Prove that the coefficient of friction is $P/\sqrt{W(2P+W)}$. 4

5. (a) If a system of coplanar forces be in equilibrium and each of the forces be rotated through the same angle, examine whether the system is also in equilibrium after the rotation. 4

(b) Find the centre of gravity of a plate in the form of a quadrant AOB of a circle of radius a. 4

6. (a) Four rods are joined together to form a parallelogram, the opposite joints are joined by strings forming the diagonals and the whole system is placed on a smooth horizontal table. Show that their tensions are in the same ratio as their lengths. 5

(b) Find the centre of gravity of the volume formed by revolving the area bounded by the parabolas $y^2=4ax$ and $x^2=4by$ about the axis of x. 3

7. (a) Prove that for a particle of mass m falling from rest under gravity from a height h above the ground, the sum of the K.E. and the P.E. of the particle is constant at every point of its path. 3

(b) A uniform hemisphere of radius a and weight W, rests with its spherical surface on a horizontal plane and a rough particle of weight W' rests on the plane surface. Show that the distance of the particle from the centre of the plane face is not grater than $\frac{3W\mu a}{8W'}$, where μ is the coefficient of friction. 5

8. (a) A particle of mass m moves in a straight line under an attractive force mn^2x towards a fixed point on the line when at a distance x from it. If it be projected with a velocity V towards the centre of force from an initial distance a from it, prove that it reaches the centre of force after a time $\frac{1}{n}\tan^{-1}\frac{na}{V}$. 5

(b) A particle moves with an acceleration which is always directed towards a fixed point O and equal to $\frac{\mu}{x}$, where x is the distance of the particle from O. If the particle starts from rest at a distance a from O then find the velocity of the particle when it is at a distance x from O. 3

9. (a) A point moves in a curve so that its tangential and normal accelerations are equal and the angular velocity of the tangent is constant. Find the curve. 3

(b) Define S.H.M. Find the equation of motion of a particle of mass m executing S.H.M. of amplitude a and find the period of oscillation. 5

10.(a) A particle is acted on by a force parallel to the axis of y where acceleration is ky and is initially projected with a velocity $a\sqrt{k}$ parallel to the axis of x at a point where $y=a$. Prove that it will describe a catenary. 5

(b) If a particle describe a rectangular hyperbola under a force which always parallel to an asymptotes. Find the law of force. 3

11.(a) Find the radial and cross radial components of velocity of a particle moving along a plane curve. 3

(b) A straight smooth tube revolves with angular velocity Ω in a horizontal plane about one extremity which is fixed. If initially the particle is at rest at a distance a from the fixed end then find the distance of the particle from the fixed end at time t. 5

12. A particle of mass m is projected into the air with velocity in a direction making an angle α with the horizontal. To find the equations of motion and solve them to obtain the path described. 5+1+1+1

Find also the time of flight taken by the particle to reach the horizontal plane again and the horizontal range covered.

Also find the angle of projection for which the horizontal range is maximum.

13.(a) One end of an elastic string is fixed at A and the other end is fastened to a heavy particle, the modulus of elasticity of the string being equal to the weight of the particle. Show that if the particle is dropped from A, it will descend a distance $(2+\sqrt{3})a$ before coming to rest. 5

(b) A particle m is attached by a light string, of length l, to a fixed point and oscillates under gravity through a small angle; find the period of its motion. 3

N.B. : *Students have to complete submission of their Answer Scripts through E-mail / Whatsapp to their own respective colleges on the same day / date of examination within 1 hour after end of exam. University / College authorities will not be held responsible for wrong submission (at in proper address). Students are strongly advised not to submit multiple copies of the same answer script.*

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